Enrollment No: _______________________ **Exam Seat No:**_______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Graph Theory

Subject Code: 4SC06GTC1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date:21/04/2017** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

Define the following terms:

a) Graph. (01)
b) Degree of vertex. (01)
c) Pendent vertex. (01)
d) Null graph. (01)
e) Simple graph. (01)
f) Complete graph. (01)
g) Circuit. (01)
h) Connected graph. (01)
i) Euler graph. (01)
j) Hamiltonian graph. (01)
k) Spanning tree. (01)
l) Binary tree. (01)
m) Branches. (01)
n) Cut set. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Let $G = (V, E)$ be a graph, where $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and correspondence between elements of V and E are $e_1 = v_1v_2,\ e_2 = v_1v_6,\ e_3 = v_6v_2,\ e_4 = v_5v_2,\ e_5 = v_5v_6,\ e_6 = v_3v_5,$ $e_7 = v_2v_2$ then represent G as graphically and give the answer of following questions (07)

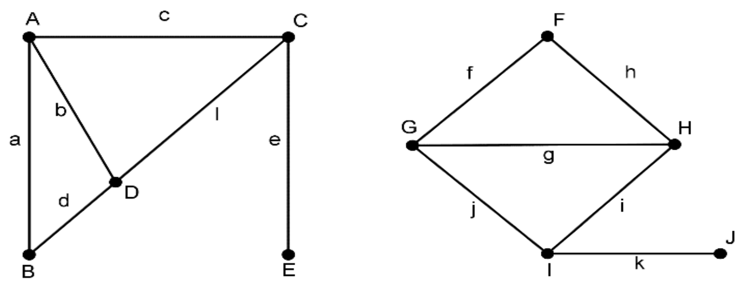
(i) Find isolated vertex of G
(ii) Find pendent vertex of G
(iii) Find even and odd vertices of G.
(iv) Verify first theorem of graph theory.
(v) Verify that number of odd vertices in graph is even.

b) State and prove first theorem of graph theory. (04)

c) Let the order of graph G be n. Among these n vertices, t vertices are of degree k and remaining vertices are of degree $k+1$ then prove that $n = \frac{t+2e}{k+1}$ where e is the number of edges in graph G (03)

Q-3 **Attempt all questions** **(14)**

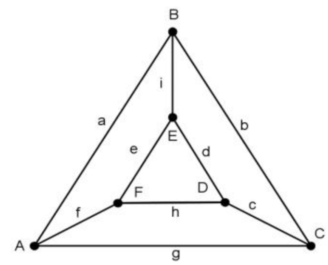
a) Define isomorphism of graphs. Show that the following graphs are isomorphic. (05)



b) Prove that number of edges in complete graph K_n is $\frac{n\,(n-1)}{2}$. (05)

c) For the adjacent graph find the following: (04)

(i) Two circuits.
(ii) Two path between A and D.
(iii) One walk between A and D but which is not path.
(iv) One spanning tree.
(v) Two cut sets.



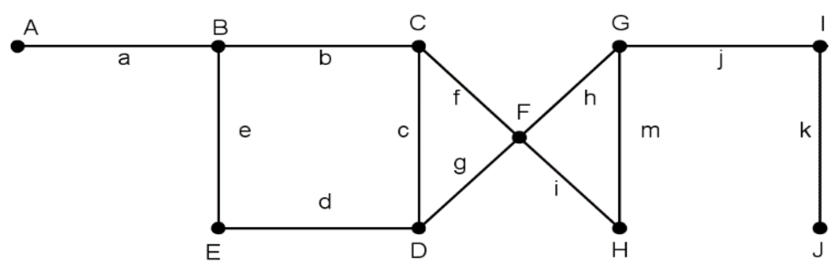
Q-4 **Attempt all questions** **(14)**

a) Explain Konisberg bridge problem.Solve it by using Euler's theorem. (07)

b) Let G be a simple graph with n vertices and k components. Then prove that G can have at most $\frac{(n-k)(n-k+1)}{2}$ number of edges. (07)

Q-5 **Attempt all questions** **(14)**

a) Define: Eccentricity. Find radius and diameter of the following graph. (07)



b) Without drawing graph check whether the graph corresponding to the adjacency matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is connected or not. (04)

c) A graph has degree sequence 1,1,2,2,2,2,4. Find number of edges of this graph and draw the graph. (03)

Q-6 **Attempt all questions** **(14)**

a) State and prove necessary and sufficient condition for disconnected graph. (07)

b) State and prove Euler's theorem. (07)

Q-7 **Attempt all questions** **(14)**

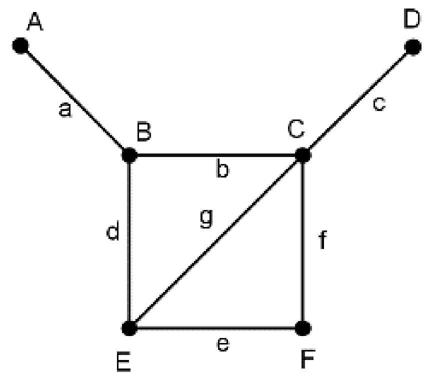
a) Define: Tree. Prove that if T is a tree with n vertices then it has precisely $n-1$ edges. (06)

b) Prove that a vertex v is a cut vertex of connected graph G if and only if there exists two vertices x and y of graph G such that every path between x and y passes through v. (05)

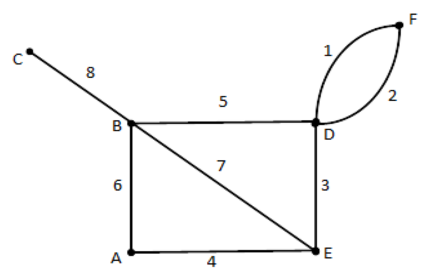
c) Prove that only complete bipartite graph which is a tree must be a star graph. (03)

Q-8 **Attempt all questions** **(14)**

a) Write adjacency, incidence and circuit matrices of the following graph: (06)



b) In usual notation verify for the following graph that $A\,B^T = B\,A^T = 0\ (mod\ 2)$ whose columns are arranged using the same order of edges. (05)



c) Define: Fusion graph. Find a fusion graph of the following graph by fusing the vertices F and H. (03)